# Precision Theory and Jet Mass Distributions

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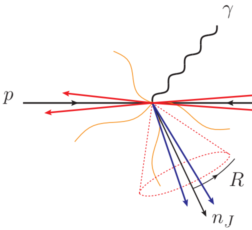
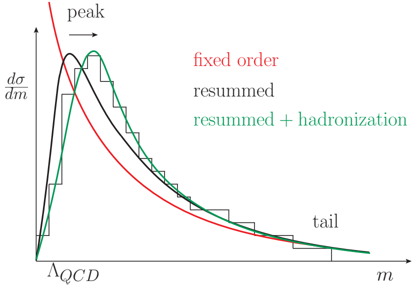
# Outline

- Jet mass
  - Resummation of large logarithms
- Soft-collinear effective theory (SCET)
  - Factorization theorem
  - Renormalization group evolution
  - Medium modification by Glauber interactions
- Conclusions and outlook

# Jet mass




$$m^2 = \Big(\sum_{i\in J} p_i\Big)^2 = \big(p_c + p_s\big)^2 \approx p_c^2 + 2p_c\cdot p_s \approx p_c^2 + 2E_J n_J \cdot p_s$$

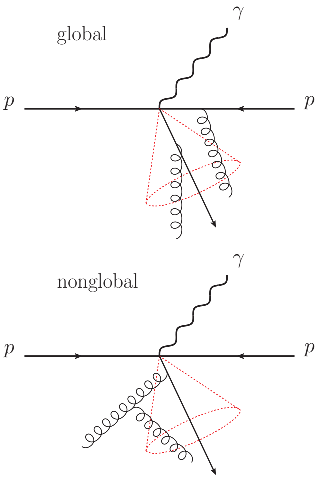



- Jet mass is a soft radiation sensitive jet substructure observable
- The infrared structure of QCD induces Sudakov logarithms
- Fixed order calculation breaks down at small jet mass $m$
- Large logarithms of the form

$$\frac{1}{m}\alpha_s^i\Big(\log^j\frac{m}{E_J}\ \text{or}\ \log^j R\Big),\quad j\le 2i-1$$

  need to be resummed
- Hadronization affects the position of the peak at small $m$
- Resummation of $\log R$ is crucial especially for jets with small radii

# Resummation precision




$$\frac{1}{m}\alpha_s^i\left(\log^j\frac{m}{E_J} \text{ or } \log^j R\right), \quad j \le 2i-1$$

- *All-order* resummation: $i = 1, \ldots \infty$
- Infrared structure of QCD allows the all-order resummation of logarithmically enhanced terms without calculating diagrams at all orders
  - leading-logarithmic (LL) accuracy: $j = 2i - 1$
  - next-to-leading-logarithmic (NLL) accuracy: $j = 2i-1, 2i-2$
  - $\ldots$
- Nonglobal logs and clustering logs appear at NNLL
  - Resummation is still an open question

# Resummation and effective field theory

## THE BASIC IDEA

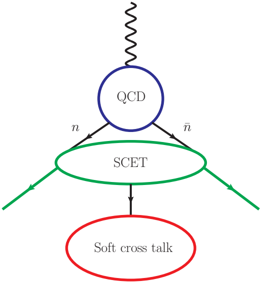
- Logarithms of *scale ratios* appear in perturbative calculations
  - Logarithms become large when scales become hierarchical

$$\log \frac{m}{E_J}, \qquad \log R = \log \frac{\text{scale 1}}{\text{scale 2}}?$$

- In effective field theories, logarithms are resummed using renormalization group evolution between characteristic scales
  - To resum *all* the logarithms we need to identify *all* the relevant scales in EFT

# Soft-Collinear Effective Theory (SCET)

- Effective field theory techniques are most useful when there is clear scale separation
- SCET separates physical degrees of freedom in QCD by a systematic expansion in power counting
  - Match SCET with QCD at the hard scale by integrating out the hard modes
  - Integrating out the off-shell modes gives collinear Wilson lines which describe the collinear radiation
  - The soft sector is described by soft Wilson lines along the jet directions



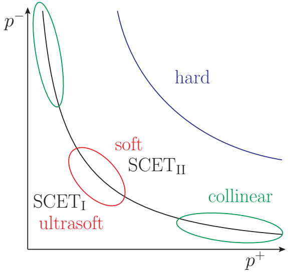

- At leading power, soft-collinear decoupling holds in the Lagrangian and it leads to the factorization of cross sections

# Power counting in SCET

- The scaling of modes in lightcone coordinates:
  $p_h : E_J(1,1,1)$, $p_c : E_J(1,\lambda^2,\lambda)$ and $p_s : E_s(1,R^2,R)$
  - $E_J$ is the hard scale which is the energy of the jet
  - $\lambda$ is the **power counting** parameter ($\lambda \approx m/E_J$)
  - $E_J\lambda$ is the jet scale which is significantly lower than $E_J$
  - Jet mass is sensitive to c-soft modes: ultrasoft modes constrained inside jets

$$E_s = E_J\frac{\lambda^2}{R^2} = \frac{m^2}{E_J R^2}$$

- QCD $= \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \cdots$ in SCET
  - Leading-power contribution in SCET is a very good approximation



# Factorization theorem

- The cross section differential in photon $p_T$, $y$, and jet mass $m$ can be first factorized as a convolution with parton distribution functions

$$\frac{d^2\sigma}{dp_T dy dm^2} = \frac{2}{p_T}\sum_{ab}\int dv dw \;\; x_1 f_a(x_1,\mu)\; x_2 f_b(x_2,\mu)\frac{d^2\hat{\sigma}}{dw dv dm^2} \;,$$

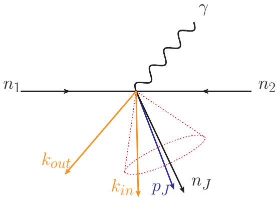
where

$$x_1 = \frac{1}{w}\frac{p_T}{E_{CM} v}e^{y}, \qquad x_2 = \frac{p_T}{E_{CM}(1-v)}e^{-y}$$

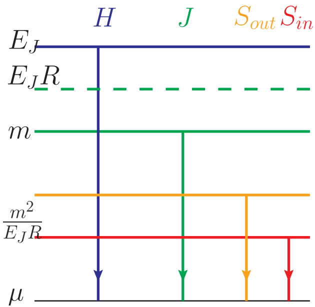
- The partonic cross section can be further factorized in SCET as a convolution of the hard, jet and soft function

$$\begin{aligned}\frac{d^2\hat{\sigma}}{dw dv dm^2} \quad &= \quad w\,\hat{\sigma}(v)\; H(p_T, v, \mu)\int dk_{in} dk_{out} dp^2 J(p^2,\mu) S(k_{in}, k_{out}, \mu) \\ &\times \delta(m^2 - p^2 - 2E_J k_{in})\delta(m_X^2 - m^2 - 2E_J k_{out})\end{aligned}$$

where $m_X^2 = (p_J + k_{in} + k_{out})^2$ is the partonic mass of the event



# Scale hierarchy and renormalization group evolution




- Each factorized piece $\mathcal{O}$ captures physics at certain characteristic scale $\mu_{\mathcal{O}}$
  - Caveat: the soft sector is multi-scaled and needs to be *refactorized*
- The renormalization group evolution between characteristic scales resums the logs of the scale ratios

$$\mu \frac{d\mathcal{O}}{d\mu} = \gamma_{\mathcal{O}} \mathcal{O}$$

  - The anomalous dimension $\gamma_{\mathcal{O}}$ can be calculated order-by-order in perturbation theory

# Resummed cross section

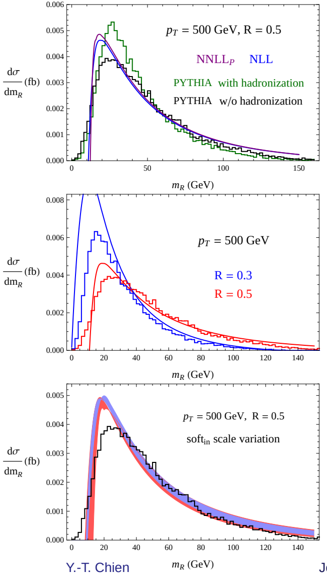
- For the $q\bar{q} \to g\gamma$ channel,

$$
\begin{aligned}
\frac{\mathrm{d}^2\hat{\sigma}_{q\bar{q}}}{\mathrm{d}w\mathrm{d}v\mathrm{d}m^2} \quad = \quad & w\hat{\sigma}_{q\bar{q}}(v)\, \exp[(4C_F+2C_A)S(\mu_h,\mu) - 4C_A S(\mu_j,\mu) + 2C_A S(\mu_{in},\mu)] \\
\times \quad & \exp[-4C_F S(\mu_{out},\mu) - 2A_H(\mu_h,\mu) + 2A_{J_g}(\mu_j,\mu)] \\
\times \quad & \exp[+2A_{S_{q\bar{q}}}(\mu_{in},\mu) + 2A_{S_{rq\bar{q}}}(\mu_{out},\mu)] \\
\times \quad & \frac{\alpha_s(\mu_h)}{\alpha_s(\mu)} \left(\frac{1}{r^2} - \beta^2\right)^{-C_F A_\Gamma(\mu_{in},\mu_{out})} \left(\frac{1}{r^2} - \frac{1}{\beta^2}\right)^{-C_F A_\Gamma(\mu_{in},\mu_{out})} \\
\times \quad & \left[\frac{\beta^2}{(1+\beta^2)^2}\right]^{-C_A A_\Gamma(\mu_{in},\mu_{out})} \left(\frac{1+r^2}{r^2}\right)^{(2C_F - C_A)A_\Gamma(\mu_{in},\mu_{out})} \\
\times \quad & (v\bar{v})^{2C_F A_\Gamma(\mu_h,\mu)} \left(\frac{p_T^2}{\mu_h^2}\right)^{-(2C_F+C_A)A_\Gamma(\mu_h,\mu)} \left(\frac{\mu_j^2}{p_T\mu_{in}}\right)^{-2C_A A_\Gamma(\mu_{in},\mu)} \\
\times \quad & H_{q\bar{q}}(p_T, v, \mu_h)\, \tilde{j}_g(\partial_{\eta_{q\bar{q}}}, \mu_j)\, \tilde{s}_{q\bar{q}}\left(\ln\frac{\mu_j^2}{p_T\mu_{in}} + \partial_{\eta_{q\bar{q}}}, \mu_{in}\right) \tilde{s}_{rq\bar{q}}(\partial_{\eta_2^s}, \mu_{out}) \\
\times \quad & \frac{1}{m_R^2(m_X^2 - m^2)} \left(\frac{m^2}{\mu_j^2}\right)^{\eta_{q\bar{q}}} \left(\frac{m_X^2 - m^2}{p_T\mu_{out}}\right)^{\eta_2^s} \frac{e^{-\gamma_E\eta_{q\bar{q}}}}{\Gamma[\eta_{q\bar{q}}]} \frac{e^{-\gamma_E\eta_2^s}}{\Gamma[\eta_2^s]}
\end{aligned}
$$

where

$$
\eta_{q\bar{q}} = 2C_A A_\Gamma(\mu_j, \mu_{s_{in}}), \qquad \eta_2^s = 4C_F A_\Gamma(\mu_{s_{out}}, \mu)
$$

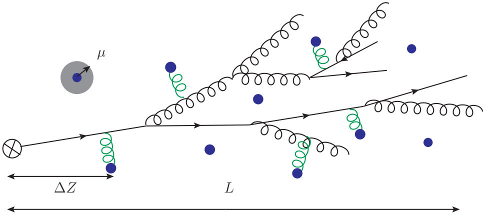
# Results




- The most precise analytic calculation of jet mass distributions to date
- Agree nicely with PYTHIA partonic calculation within theoretical uncertainty
  - Comparison with data will be performed
- Hadronization effect plays a role as shown in PYTHIA simulations
  - Analytic study of nonperturbative soft matrix element will be included
- Jet radius dependence correctly captured

# Multiple scattering in a medium

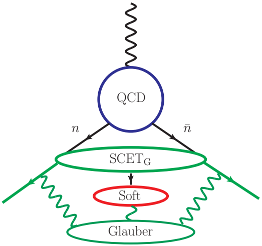
- Coherent multiple scattering and induced bremsstrahlung are the qualitatively new ingredients in the medium parton shower
- Interplay between several characteristic scales:
  - Debye screening scale $\mu$
  - Parton mean free path $\lambda$
  - Radiation formation time $\tau$
- From thermal field theory and lattice QCD calculations, an ensemble of quasi particles with debye screened potential and thermal masses is a reasonable parameterization of the medium properties




$$\frac{1}{\sigma_{el}}\frac{d\sigma_{el}}{d^2q_\perp} = \frac{\mu^2}{\pi(q_\perp^2+\mu^2)^2}$$

# SCET with Glauber gluons ($\mathrm{SCET_G}$)

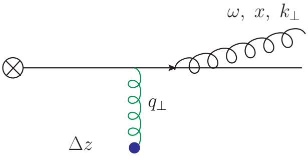
- Glauber gluon is the relevant mode for medium interactions
- $\mathrm{SCET_G}$ was constructed from SCET bottom up (Idilbi et al, Vitev et al)
    - Glauber momentum scales as $p_G : Q(\lambda^2, \lambda^2, \lambda)$
    - Glauber gluons are off-shell modes providing momentum transfer transverse to the jet direction
    - Glauber gluons are treated as background fields generated from the color charges in the QGP
    - Glauber gluons interact with both the collinear and the soft modes
- Given a medium model, we can use $\mathrm{SCET_G}$ to consistently couple the medium to jets



# Medium-induced splitting

- The hierarchy between $\tau$ and $\lambda$ determines the degree of coherence between multiple scatterings

$$\tau = \frac{x\,\omega}{(q_\perp - k_\perp)^2} \quad \text{v.s.} \quad \lambda$$




- Medium-induced splitting functions were calculated using $\mathrm{SCET_G}$ (Ovanesyan et al)

$$\frac{dN^{med}_{q\to qg}}{dxd^2k_\perp} = \frac{C_F\alpha_s}{\pi^2}\frac{1}{x}\int_0^L \frac{d\Delta z}{\lambda}\int d^2q_\perp \frac{1}{\sigma_{el}}\frac{d\sigma_{el}}{d^2q_\perp}\frac{2k_\perp\cdot q_\perp}{k_\perp^2(q_\perp-k_\perp)^2}\left[1-\cos\left(\frac{(q_\perp-k_\perp)^2\Delta z}{x\omega}\right)\right]$$

- $\frac{dN^{med}}{dxd^2k_\perp}$ $\to$ finite as $k_\perp \to 0$: the Landau-Pomeranchuk-Migdal effect

# NLL jet mass function

- At NLL accuracy, we can define a jet-by-jet jet mass function $J_M(m^2, \mu)$ which captures all the soft-collinear radiation

$$J_M(m^2, \mu) = \int dp^2 dk J(p^2, \mu) S_{in}(k, \mu) \delta(m^2 - p^2 - 2E_J k)$$

- Medium-induced splitting functions can be used to calculate the modification of $J_M(m^2, \mu)$ as a power correction

$$J_M^i(m^2, \mu) = \sum_{j,k} \int_{PS} dx dk_\perp \mathcal{P}_{i \to jk}(x, k_\perp) \delta(m^2 - M^2(x, k_\perp))$$

- The full jet mass distribution can be calculated by weighting the jet mass functions with jet cross sections

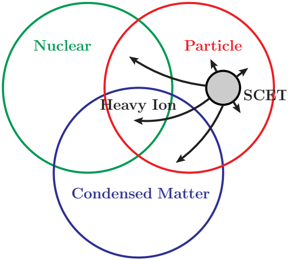
$$\frac{d\sigma}{dm^2} = \sum_{i=q,g} \int_{PS} dp_T dy \frac{d\sigma^i}{dp_T dy} \frac{J_M^i(m^2, \mu)}{J_{un}^i(\mu)}$$

- Results: stay tuned before Hard Probes 2016

# Conclusions and outlooks

- Jet mass in proton and heavy ion collisions can be calculated within the same framework
  - Promising agreement with PYTHIA in pp and phenomenological applications
- Expect the modification of jet mass to be a combination of cross section suppression and jet-by-jet broadening
- Work in progress and future work
  – Examine the Glauber-soft interactions
  – Calculate jet mass function modifications
  – Construct SCET at finite temperature
  – Study hadronization in the medium

# Conclusions and outlooks




- The physics of heavy ion collisions is a multi-disciplinary subject
- The study of jet quenching is a unique opportunity to probe non-perturbative QCD physics with perturbative objects
- Effective field theory techniques can make important contributions